7.3 Solving Linear Systems by Linear Combinations

Goals • Use linear combinations to solve a system of linear equations.
• Model a real-life problem using a system of linear equations.

VOCABULARY

Linear combination A linear combination of two equations is an equation obtained by adding one of the equations (or a multiple of one of the equations) to the other equation.

Example 1 *Using Addition*

Solve the linear system. $7x + 2y = -6$ **Equation 1**

$5x - 2y = 6$ **Equation 2**

Solution

The equations are already arranged with like terms in columns.

The coefficients for y are already opposites.

Add the equations to get an equation in one variable.

$7x + 2y = -6$ **Write Equation 1.**

$5x - 2y = 6$ **Write Equation 2.**

$12x = 0$ **Add equations.**

$x = 0$ **Solve for x.**

Substitute 0 for x in the first equation and solve for y.

$7(0) + 2y = -6$ **Substitute for x.**

$y = -3$ **Solve for y.**

Check that $(0, -3)$ is a solution by substituting 0 for x and -3 for y in each of the original equations.

Answer The solution is $(0, -3)$.

Example 2 *Using Multiplication First*

Solve the linear system. $3x - 5y = 15$ **Equation 1**

$2x + 4y = -1$ **Equation 2**

Solution

The equations are already arranged. You can get the coefficients of x to be opposites by multiplying the first equation by 2 and the second equation by -3.

$3x - 5y = 15$ **Multiply by** 2. → $6x - 10y = 30$

$2x + 4y = -1$ **Multiply by** (-3). → $-6x - 12y = 3$

Add the equations and solve for y. $-22y = 33$

$y = -1.5$

Substitute -1.5 for y in the second equation and solve for x.

$2x + 4y = -1$ **Write Equation 2.**

$2x + 4(-1.5) = -1$ **Substitute for y.**

$2x - 6 = -1$ **Simplify.**

$x = 2.5$ **Solve for x.**

Answer The solution is $(2.5, -1.5)$.

Checkpoint **Use linear combinations to solve the system of linear equations.**

1. $4x + y = -4$ $-4x + 2y = 16$ $(-2, 4)$	**2.** $7x + y = 2$ $5x + 2y = 4$ $(0, 2)$

Example 3 *Arranging Like Terms in Columns*

Solve the linear system. $3y = -6 - 4x$ **Equation 1**

$7x + 3y = -15$ **Equation 2**

Solution

First arrange the equations.

$4x + 3y = -6$ **Rearrange Equation 1.**

$7x + 3y = -15$ **Write Equation 2.**

You can get the coefficients of y to be opposites by multiplying the second equation by -1.

$4x + 3y = -6$

$7x + 3y = -15$ **Multiply by** -1.

$4x + 3y = -6$

$-7x - 3y = 15$

Add the equations and solve for x.

$-3x = 9$

$x = -3$

Substitute -3 for x in the second equation and solve for y.

$7x + 3y = -15$ **Write Equation 2.**

$7(-3) + 3y = -15$ **Substitute for** x.

$-21 + 3y = -15$ **Simplify.**

$y = 2$ **Solve for** y.

Answer The solution is $(-3, 2)$.

✔ ***Checkpoint*** **Use linear combinations to solve the system of linear equations.**

3. $x - 3y - 8 = 0$ $4y = 11 - 3x$ $(5, -1)$	**4.** $6x - 23 = -5y$ $9x + 32 = 2y$ $(-2, 7)$